

Cambridge International AS & A Level

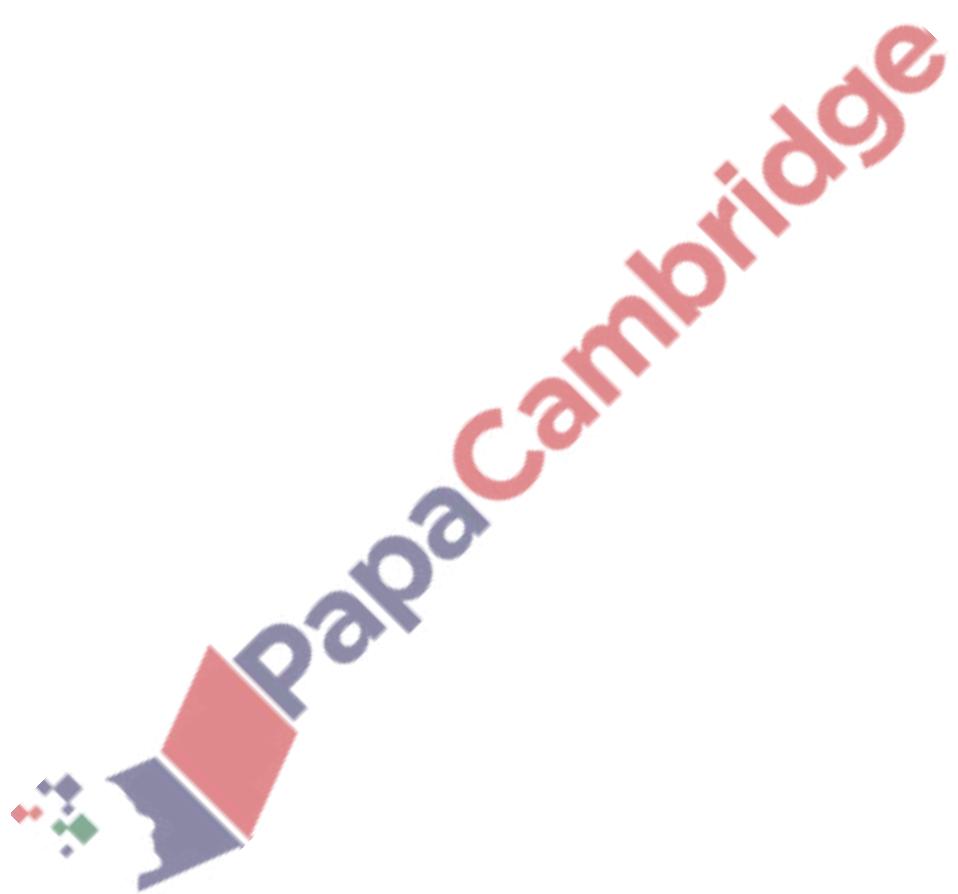
MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS



Chapter 3

Trigonometry



81. 9709_s20_qp_31 Q: 3

Express the equation $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$ as a quadratic equation in $\tan \theta$, and hence solve the equation for $0^\circ \leq \theta \leq 180^\circ$. [6]

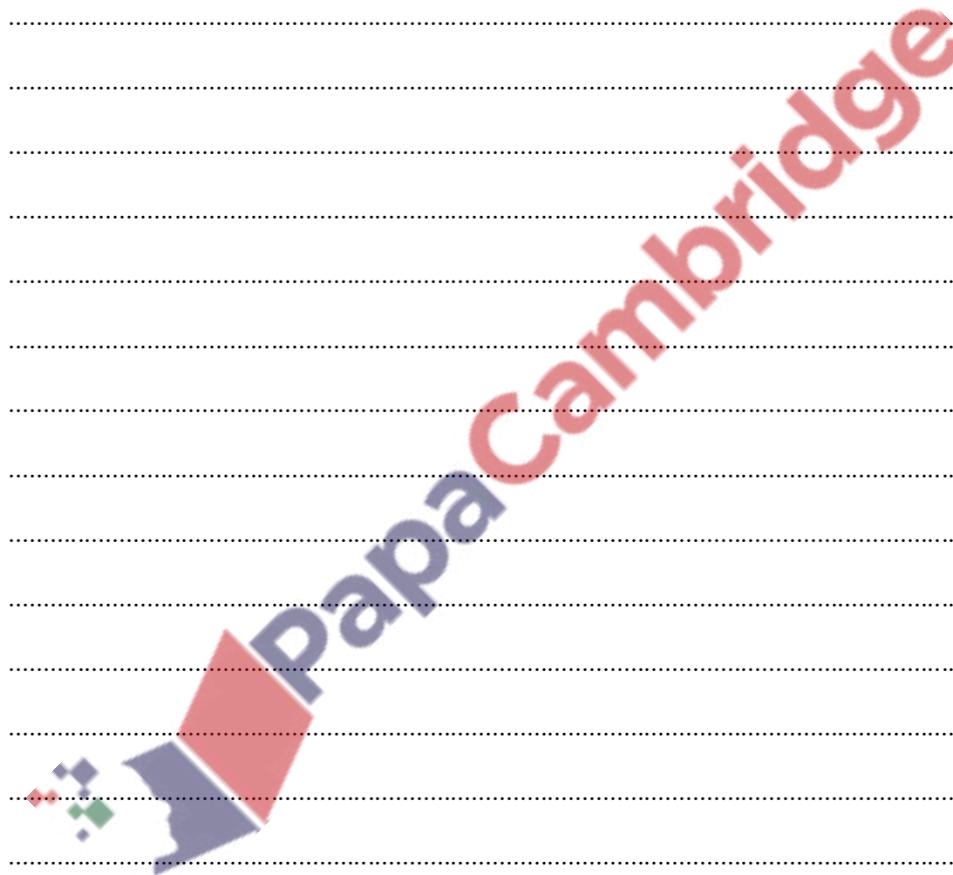
A large, diagonal watermark is positioned across the page. The watermark features the word "Papacambridge" in a stylized, slanted font. The letters are primarily red, with the "P" and "C" partially overlaid by a blue shape resembling a pencil or a brush. Below the main text, there is a small graphic element consisting of several small, colorful shapes: a green triangle, a blue circle, and a red square, arranged in a cluster.

82. 9709_s20_qp_32 Q: 5

- (a) Express $\sqrt{2} \cos x - \sqrt{5} \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: "Papa" is red, and "Cambridge" is blue. Below the text is a graphic of a 3D eraser. The eraser is primarily red with a blue base. It has a grid pattern on its side and a small green and red decorative element at the bottom left corner.

- (b) Hence solve the equation $\sqrt{2} \cos 2\theta - \sqrt{5} \sin 2\theta = 1$, for $0^\circ < \theta < 180^\circ$. [4]



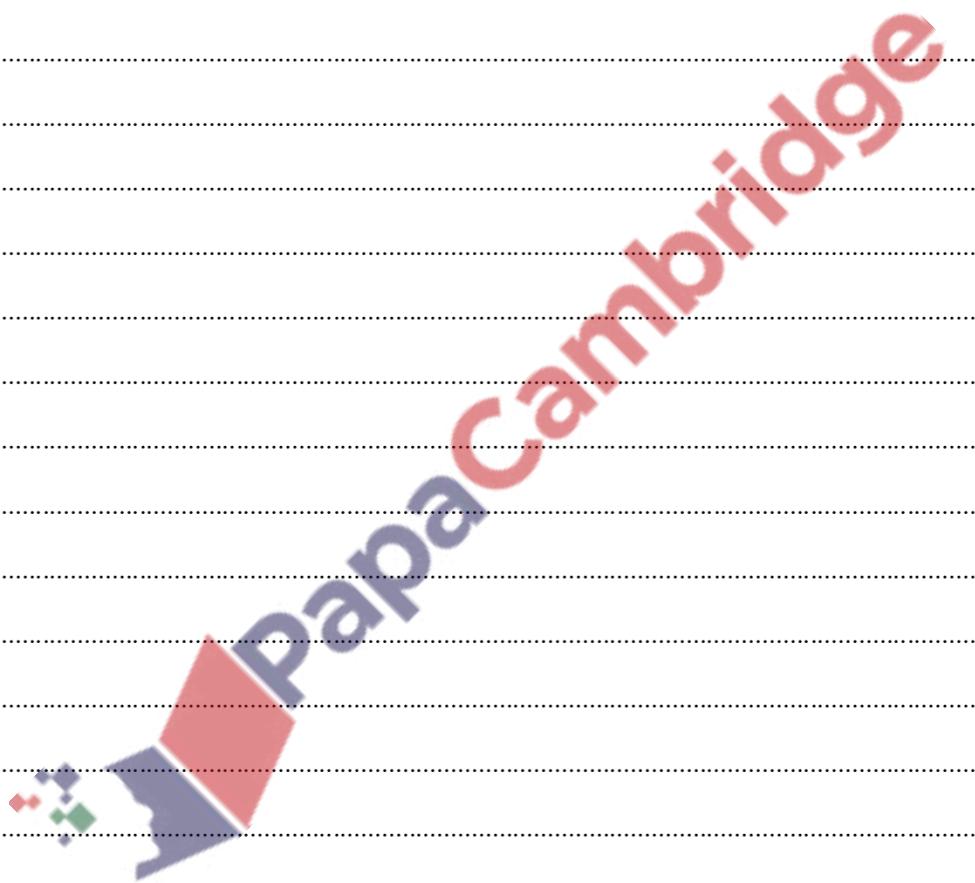
83. 9709_s20_qp_33 Q: 5

By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 90^\circ$.

[6]





84. 9709_w20_qp_31_Q: 6

- (a) Express $\sqrt{6} \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a stylized font where the letters are slanted upwards from left to right. The letters are primarily red, with some blue used for the 'P' and 'C'. Below the text is a graphic of a yellow eraser with a red top and a blue bottom, with small green and blue diamond patterns at its base.

- (b) Hence solve the equation $\sqrt{6} \cos \frac{1}{3}x + 3 \sin \frac{1}{3}x = 2.5$, for $0^\circ < x < 360^\circ$. [4]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the brand name "PapaCambridge" in a stylized, red, serif font. A red pencil is integrated into the letter "P", with its eraser end pointing towards the bottom left and its tip pointing towards the top right. Below the pencil, there is a small cluster of colorful geometric shapes, including red, green, and blue squares and dots.

85. 9709_w20_qp_32 Q: 4

- (a) Show that the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$ can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0.$$

[3]

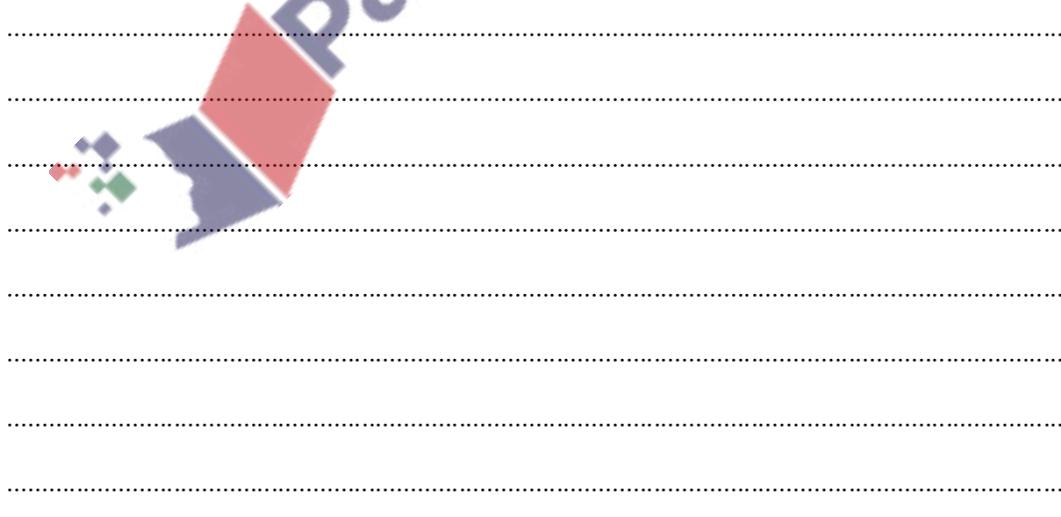


- (b) Hence solve the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$, for $0^\circ < \theta < 180^\circ$. [3]

86. 9709 - m19 - qp - 32 Q: 3

- (i) Given that $\sin(\theta + 45^\circ) + 2 \cos(\theta + 60^\circ) = 3 \cos \theta$, find the exact value of $\tan \theta$ in a form involving surds. You need not simplify your answer. [4]

- (ii) Hence solve the equation $\sin(\theta + 45^\circ) + 2 \cos(\theta + 60^\circ) = 3 \cos \theta$ for $0^\circ < \theta < 360^\circ$. [2]



87. 9709_s19_qp_31 Q: 4

By first expressing the equation $\cot \theta - \cot(\theta + 45^\circ) = 3$ as a quadratic equation in $\tan \theta$, solve the equation for $0^\circ < \theta < 180^\circ$. [6]

88. 9709_s19_qp_31 Q: 6

- (i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]



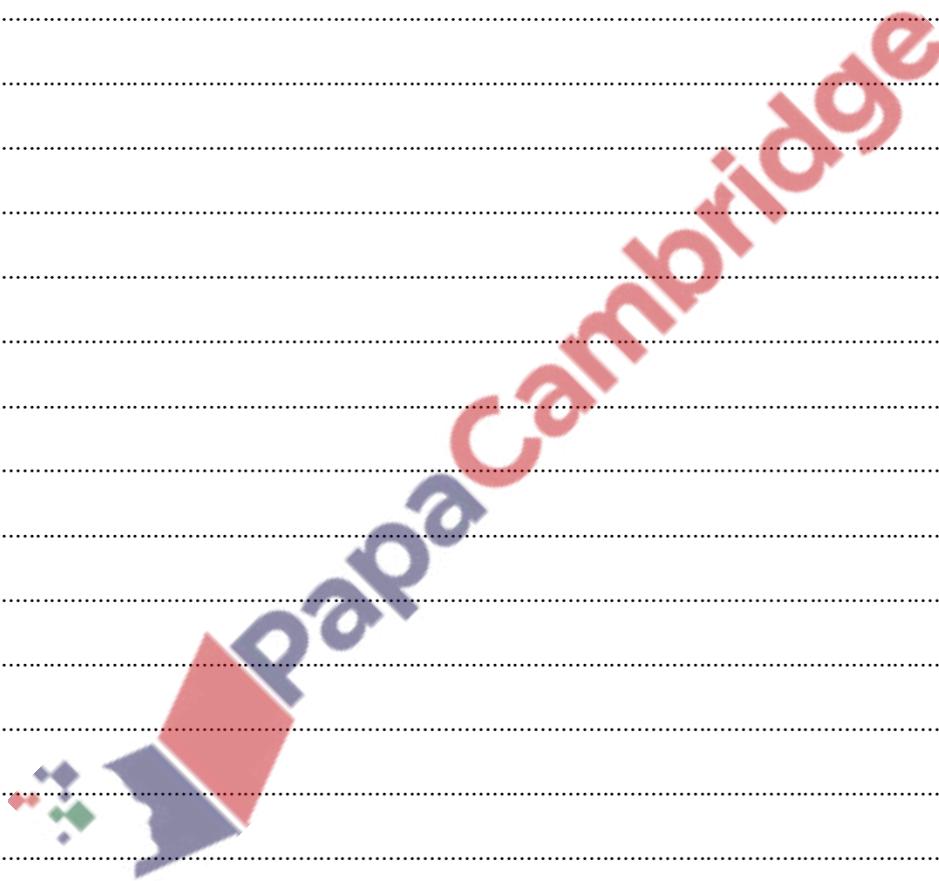
- (ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx$. [4]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "PapaCambridge" in a bold, sans-serif font. The letters are colored in a light red or maroon hue. Below the text, there is a graphic element consisting of a red pencil standing upright next to a blue eraser. The eraser has some small, colorful geometric shapes (green, red, blue) scattered around its base.

89. 9709_s19_qp_32 Q: 3

Showing all necessary working, solve the equation $\cot 2\theta = 2 \tan \theta$ for $0^\circ < \theta < 180^\circ$.

[5]

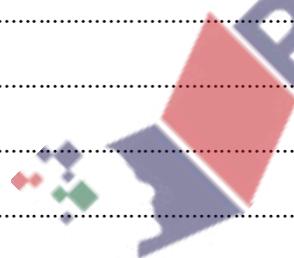


90. 9709_w19_qp_32 Q: 4

- (i) Express $(\sqrt{6}) \sin x + \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 3 decimal places. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font, with "Papa" in blue and "Cambridge" in red. A stylized graphic of a pencil tip is integrated into the letter "P", with the body of the pencil extending downwards and to the left. The pencil is primarily blue with a red eraser at the top. At the base of the pencil, there is a small cluster of geometric shapes including triangles and squares in shades of green, blue, and red.

- (ii) Hence solve the equation $(\sqrt{6}) \sin 2\theta + \cos 2\theta = 2$, for $0^\circ < \theta < 180^\circ$. [4]



91. 9709_s18_qp_31 Q: 2

- (i) Given that $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, find the exact value of $\tan x$. [4]

- (ii) Hence solve the equation $\sin(x - 60^\circ) = 3 \cos(x - 45^\circ)$, for $0^\circ < x < 360^\circ$. [2]

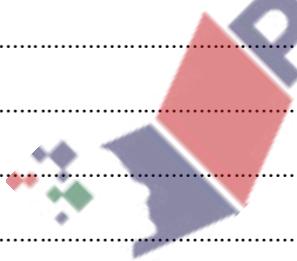


A decorative header at the top of the page features a cluster of colorful geometric shapes in the top-left corner, including red, green, blue, and yellow triangles and circles. The rest of the header is filled with five horizontal dotted lines.

92. 9709_s18_qp_32 Q: 2

Showing all necessary working, solve the equation $\cot \theta + \cot(\theta + 45^\circ) = 2$, for $0^\circ < \theta < 180^\circ$. [5]

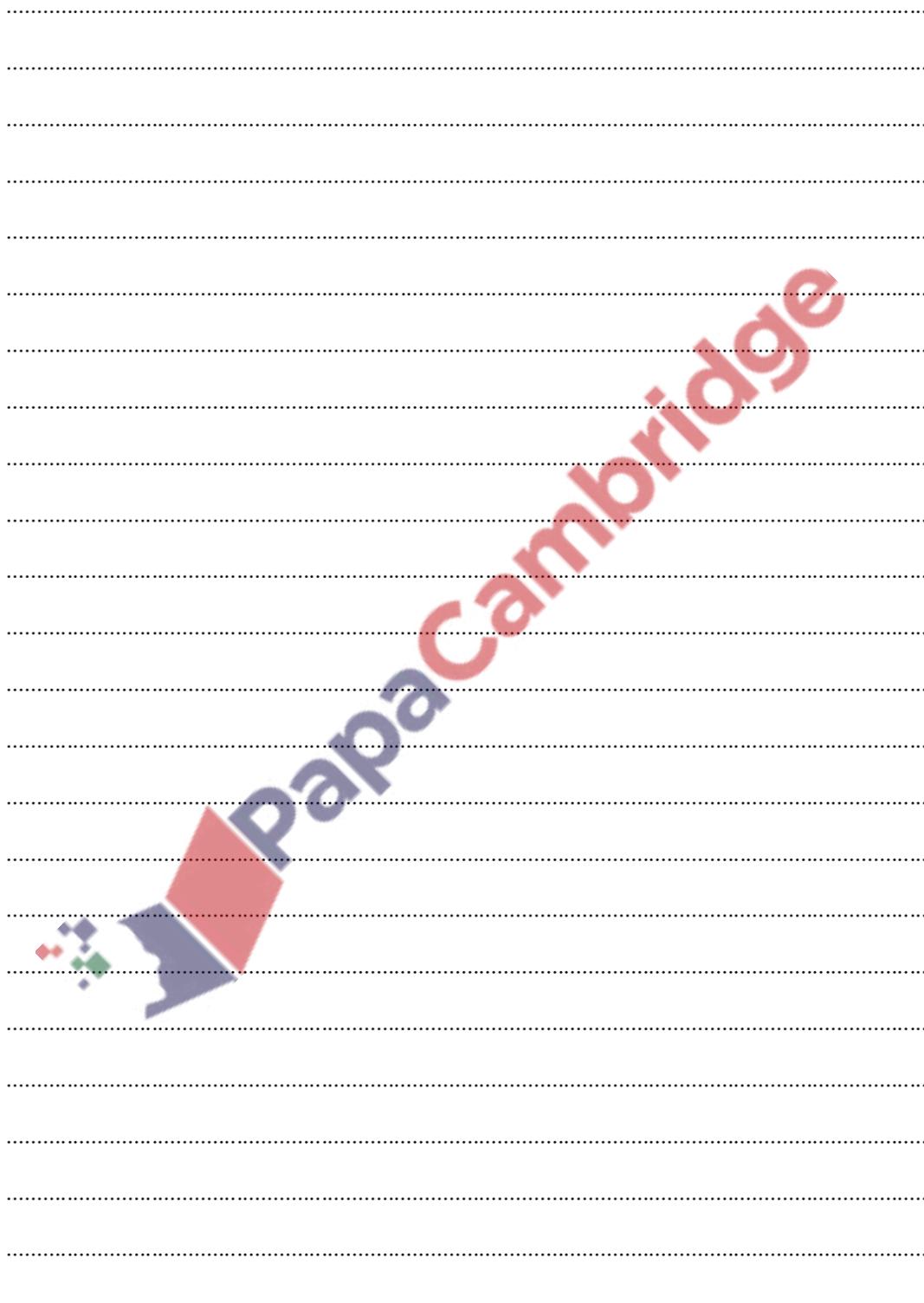
A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font, with "Papa" in blue and "Cambridge" in red. Below the text is a stylized graphic of a rocket ship or missile, oriented diagonally upwards from the bottom left. The rocket has a white body with a red nose cone and a blue base. Small colorful dots (blue, green, red) are scattered near the base of the rocket.



93. 9709_s18_qp_33 Q: 5

- (i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \frac{3}{4} \sin^2 2x. \quad [4]$$

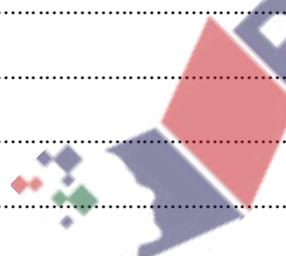


(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \frac{2}{3},$$

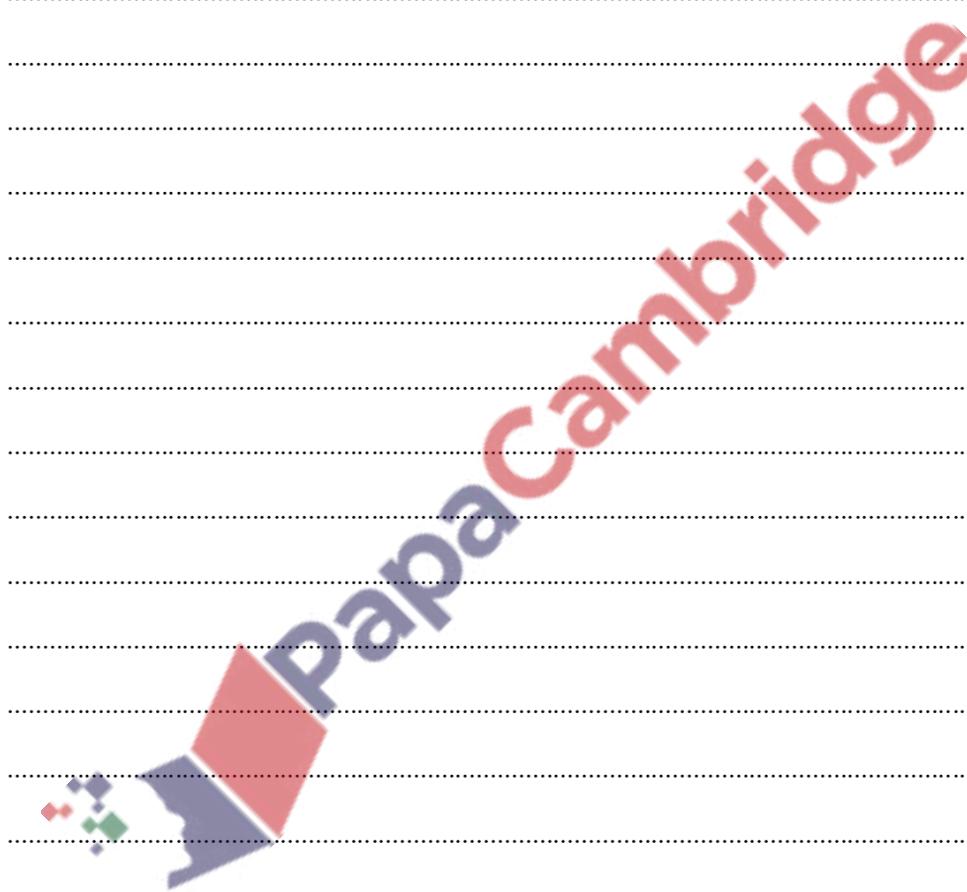
for $0^\circ < x < 180^\circ$.

[4]



94. 9709_w18_qp_31 Q: 6

- (i) Show that the equation $(\sqrt{2}) \operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R \sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]



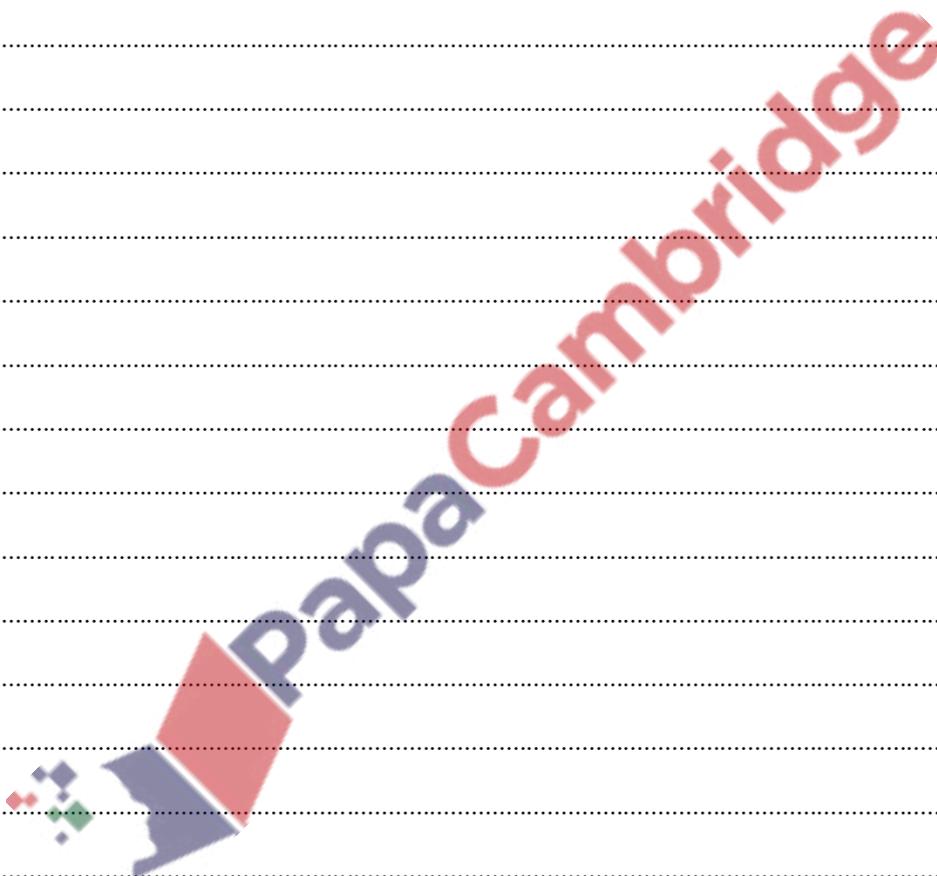
- (ii) Hence solve the equation $(\sqrt{2}) \operatorname{cosec} x + \cot x = \sqrt{3}$, for $0^\circ < x < 180^\circ$. [4]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: red for "Papa", blue for "Cam", and green for "bridge". Below the text is a graphic element consisting of several overlapping, rounded rectangular shapes in red, blue, and green, arranged in a way that suggests depth or a stylized logo.

95. 9709_w18_qp_32_Q: 2

Showing all necessary working, solve the equation $\sin(\theta - 30^\circ) + \cos \theta = 2 \sin \theta$, for $0^\circ < \theta < 180^\circ$.

[4]



96. 9709_m17_qp_32_Q: 4

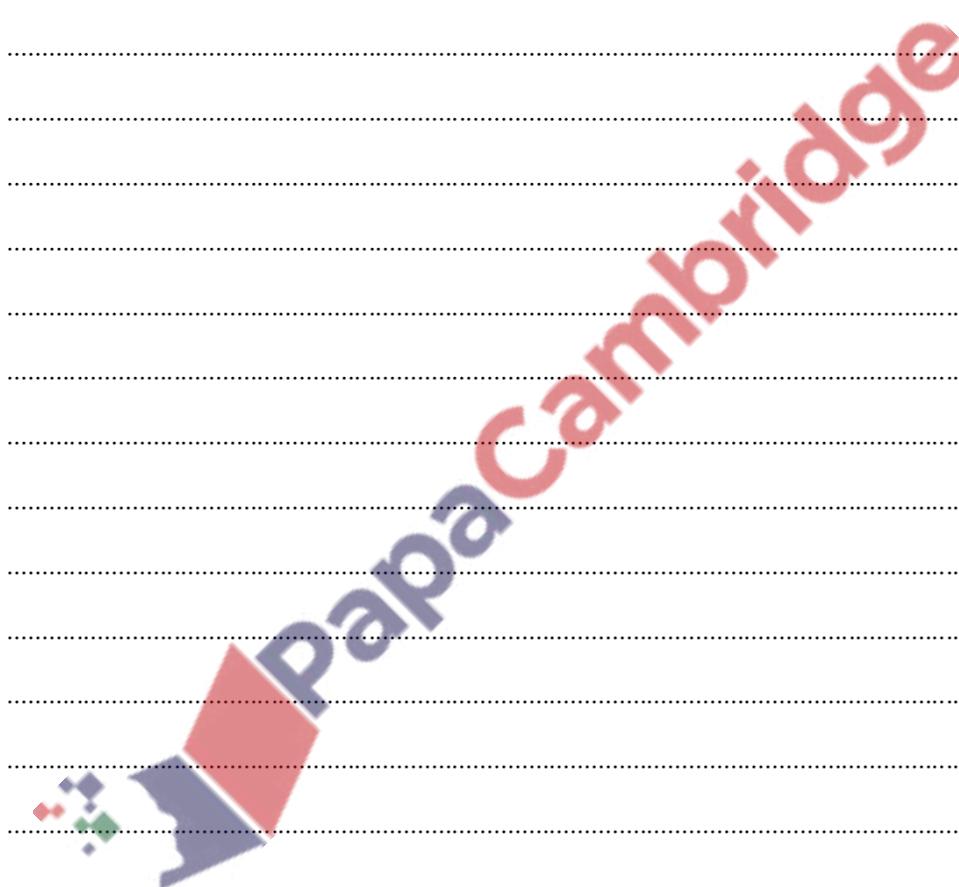
- (i) Express $8 \cos \theta - 15 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8 \cos 2x - 15 \sin 2x = 4,$$

for $0^\circ < x < 180^\circ$.

[4]



97. 9709_s17_qp_31 Q: 8

- (i) By first expanding $2 \sin(x - 30^\circ)$, express $2 \sin(x - 30^\circ) - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

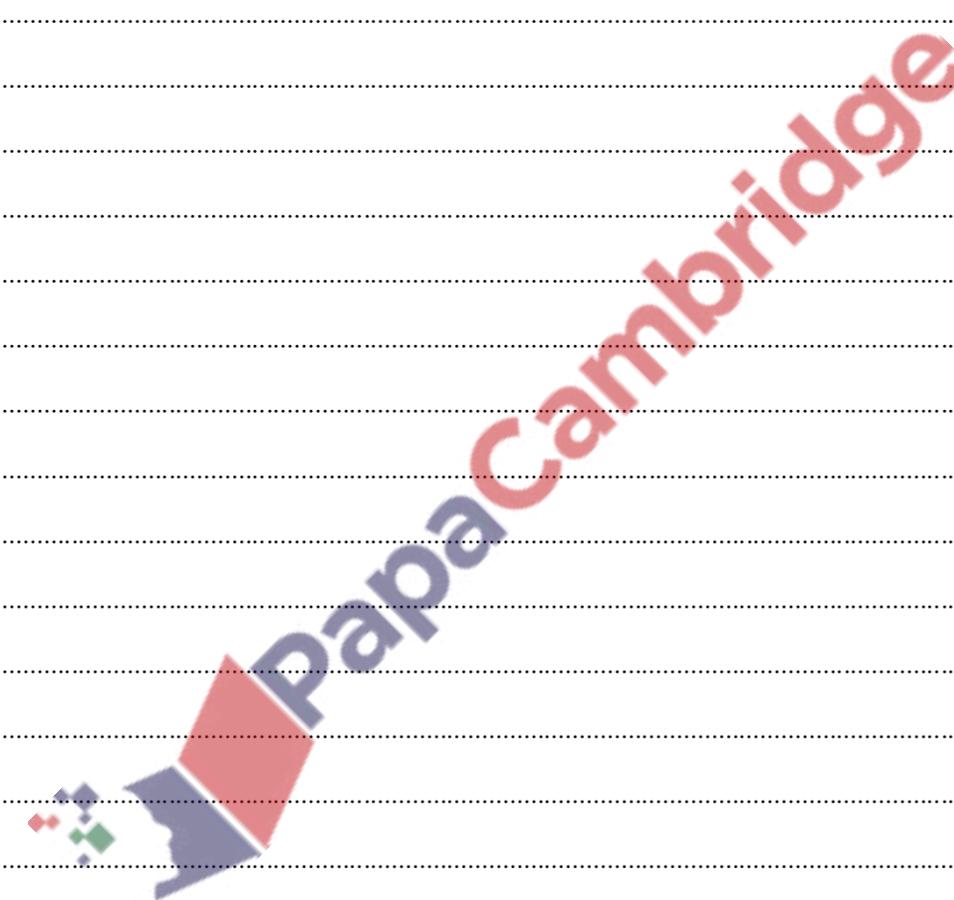
A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the brand name "Papacambridge" in a bold, sans-serif font. The letters are colored in a gradient of red and blue. Below the text, there is a graphic element consisting of a stylized pen or marker tip in red and blue, with small colored dots (red, green, blue) trailing off from its tip.

(ii) Hence solve the equation

$$2 \sin(x - 30^\circ) - \cos x = 1,$$

for $0^\circ < x < 180^\circ$.

[3]



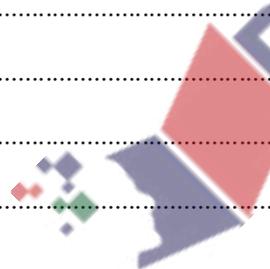
98. 9709_s17_qp_32 Q: 3

- (i) Express the equation $\cot \theta - 2 \tan \theta = \sin 2\theta$ in the form $a \cos^4 \theta + b \cos^2 \theta + c = 0$, where a , b and c are constants to be determined. [3]

- (ii) Hence solve the equation $\cot \theta - 2 \tan \theta = \sin 2\theta$ for $90^\circ < \theta < 180^\circ$. [2]

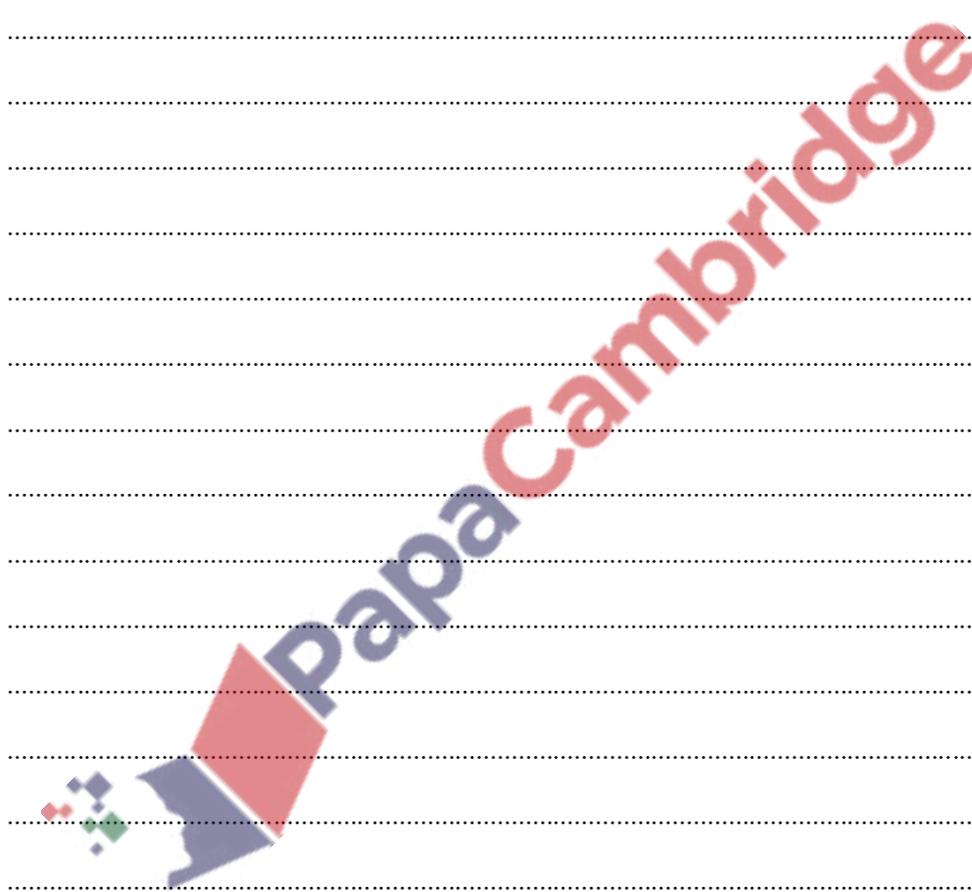
99. 9709_s17_qp_33 Q: 1

Prove the identity $\frac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$. [3]

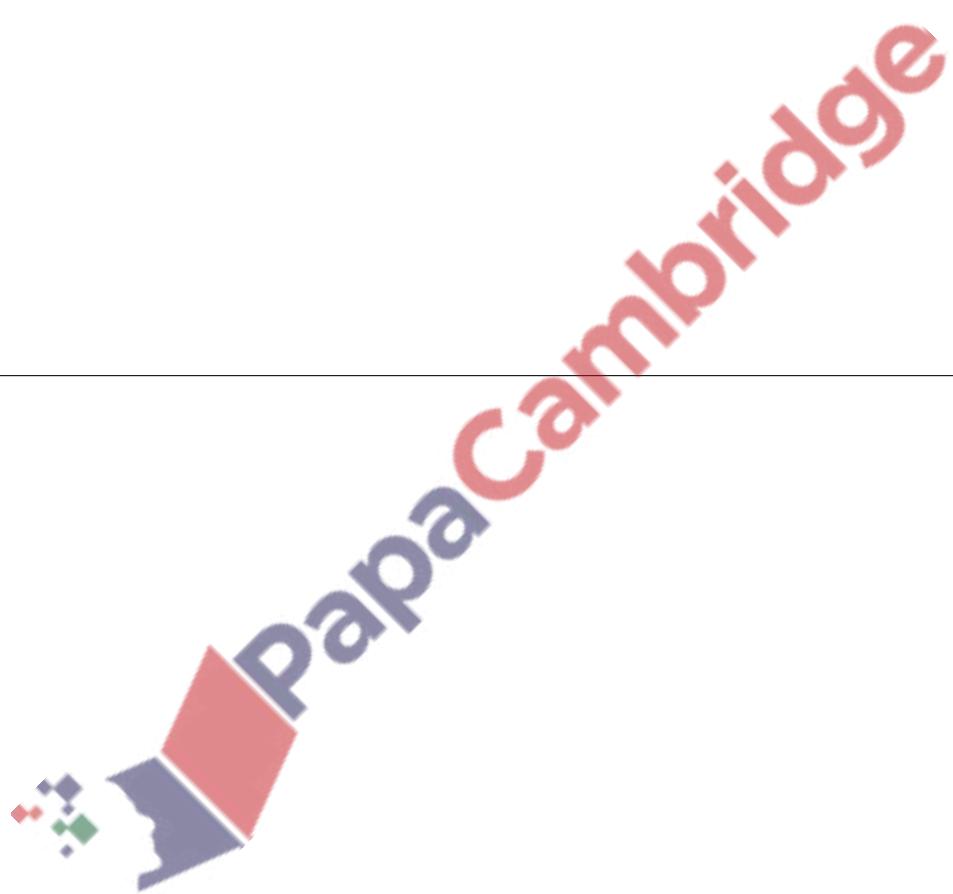


100. 9709_w17_qp_31 Q: 4

- (i) Prove the identity $\tan(45^\circ + x) + \tan(45^\circ - x) \equiv 2 \sec 2x$. [4]

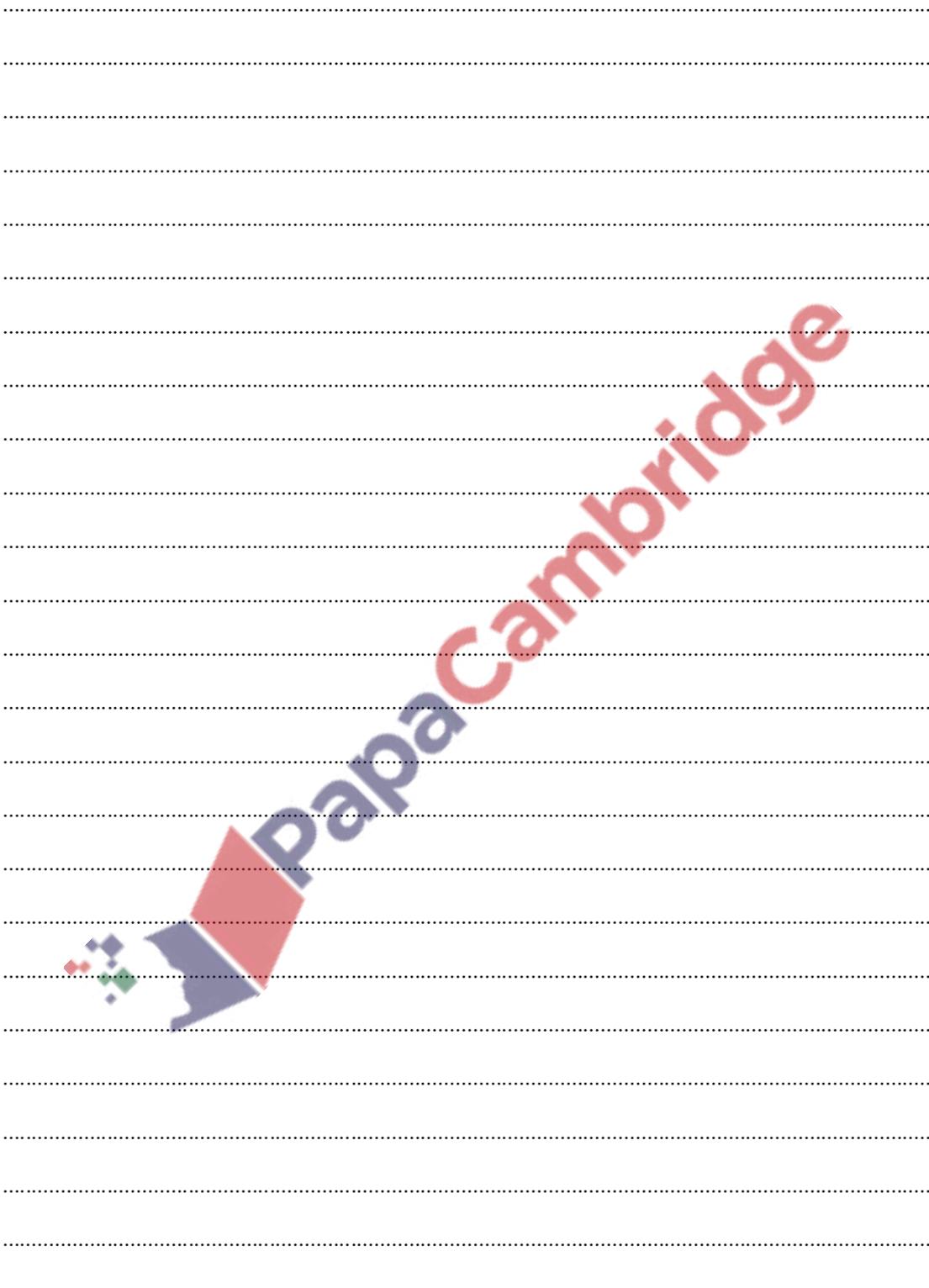


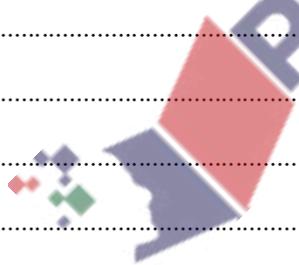
- (ii) Hence sketch the graph of $y = \tan(45^\circ + x) + \tan(45^\circ - x)$ for $0^\circ \leq x \leq 90^\circ$. [3]



101. 9709_w17_qp_32 Q: 3

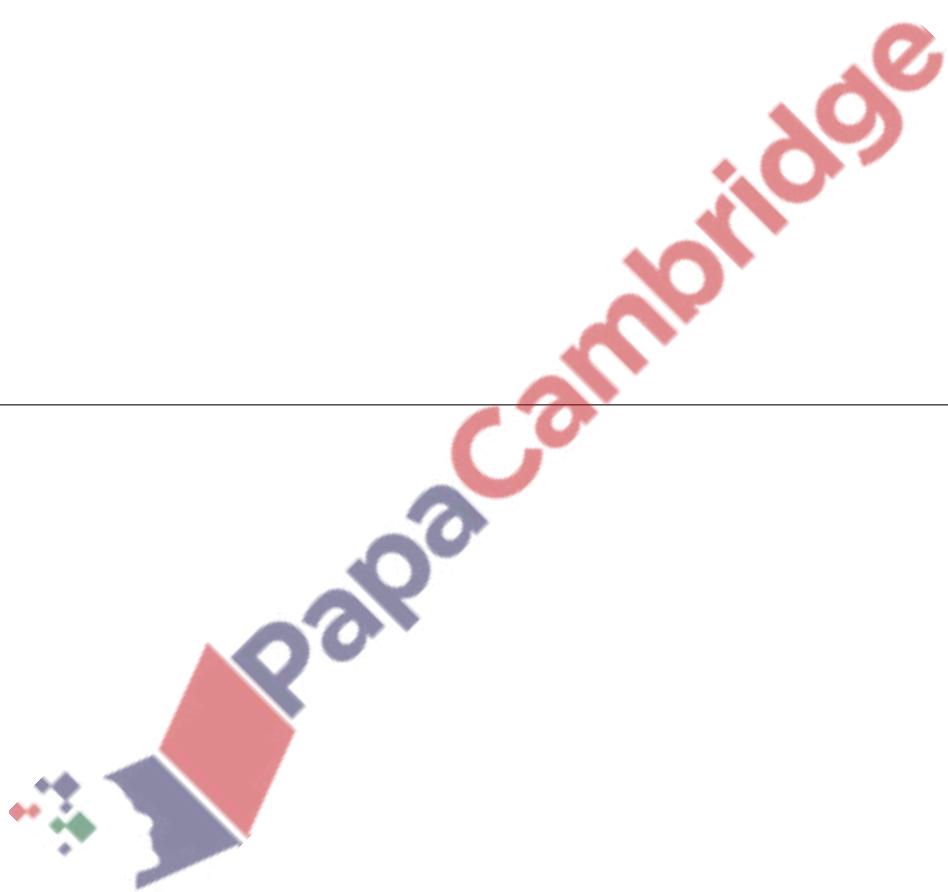
By expressing the equation $\tan(\theta + 60^\circ) + \tan(\theta - 60^\circ) = \cot \theta$ in terms of $\tan \theta$ only, solve the equation for $0^\circ < \theta < 90^\circ$. [5]





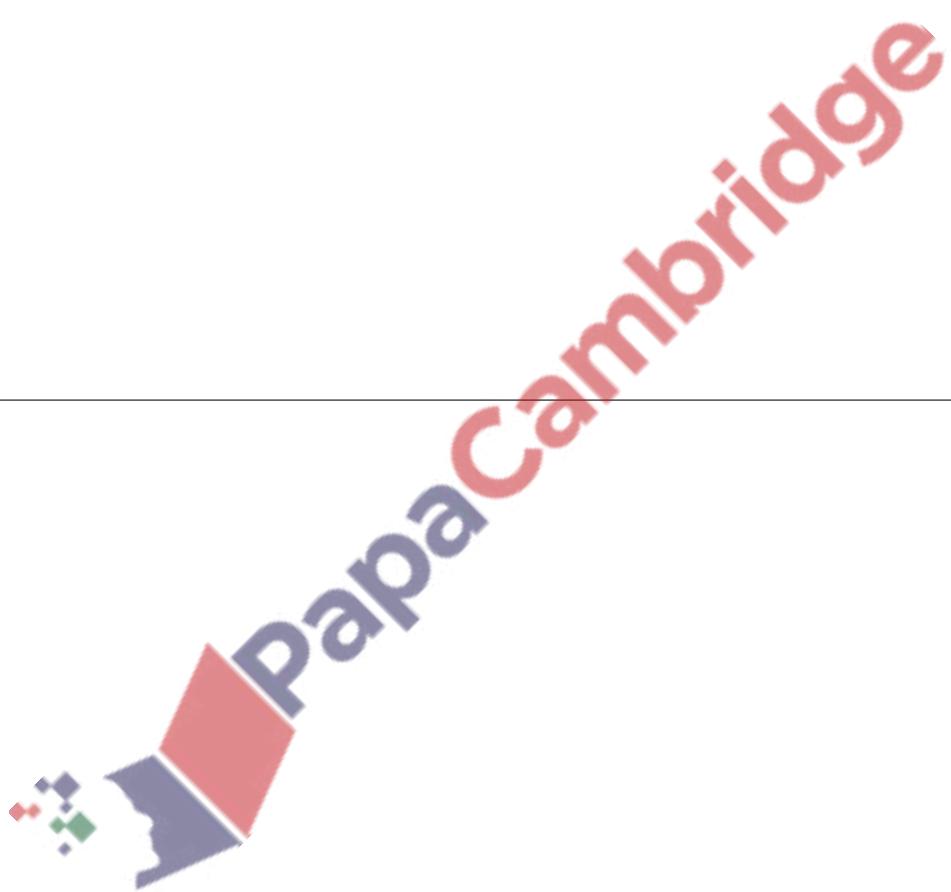
102. 9709_m16_qp_32 Q: 2

Express the equation $\tan(\theta + 45^\circ) - 2 \tan(\theta - 45^\circ) = 4$ as a quadratic equation in $\tan \theta$. Hence solve this equation for $0^\circ \leq \theta \leq 180^\circ$. [6]



103. 9709_s16_qp_31 Q: 3

By expressing the equation $\operatorname{cosec} \theta = 3 \sin \theta + \cot \theta$ in terms of $\cos \theta$ only, solve the equation for $0^\circ < \theta < 180^\circ$. [5]



104. 9709_s16_qp_32 Q: 5

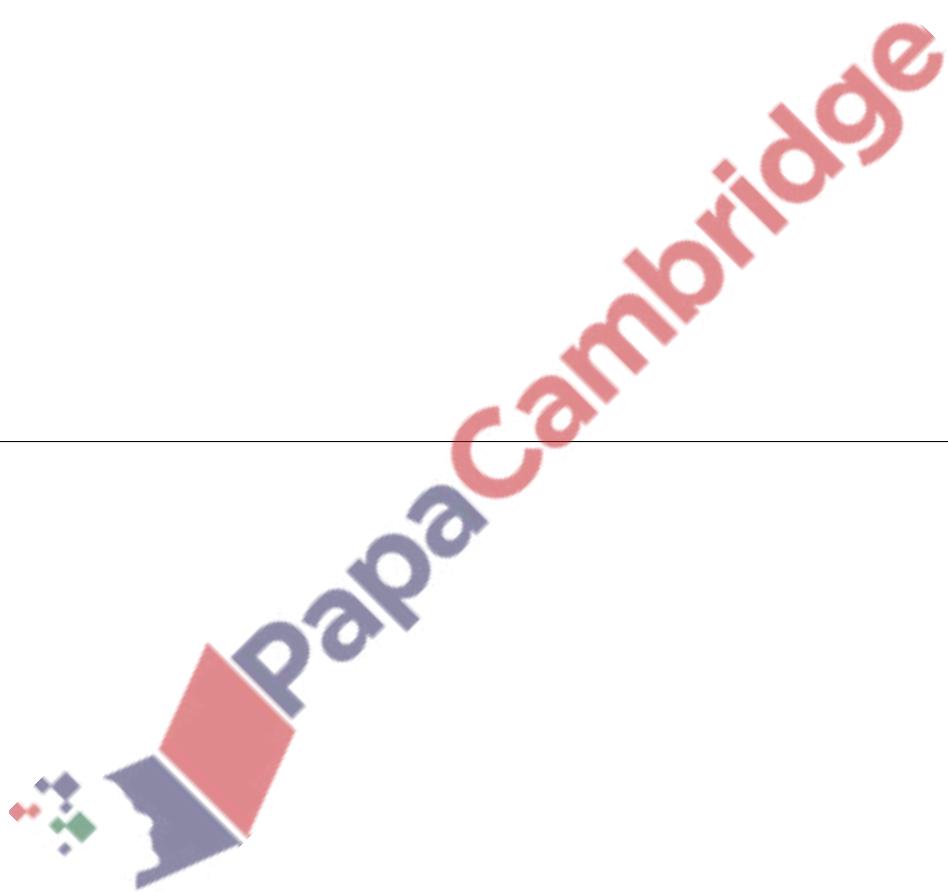
(i) Prove the identity $\cos 4\theta - 4 \cos 2\theta \equiv 8 \sin^4 \theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4 \cos 2\theta + 3,$$

for $0^\circ \leq \theta \leq 360^\circ$.

[4]



105. 9709_s16_qp_33 Q: 3

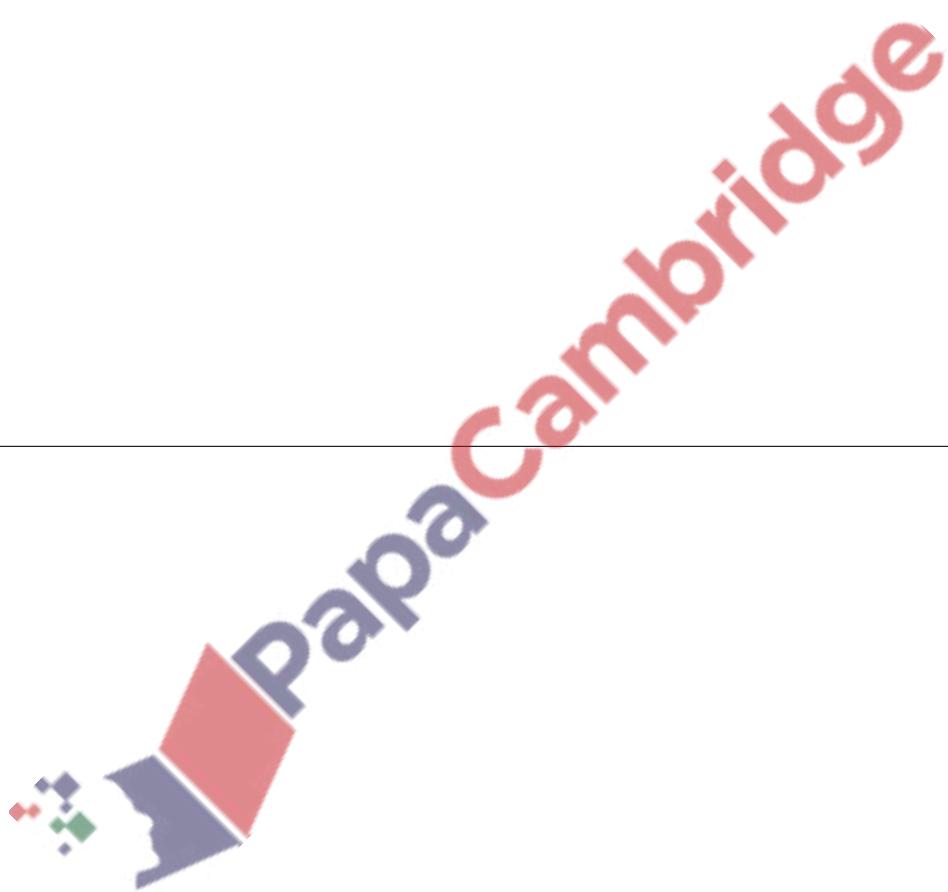
- (i) Express $(\sqrt{5}) \cos x + 2 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$(\sqrt{5}) \cos \frac{1}{2}x + 2 \sin \frac{1}{2}x = 1.2,$$

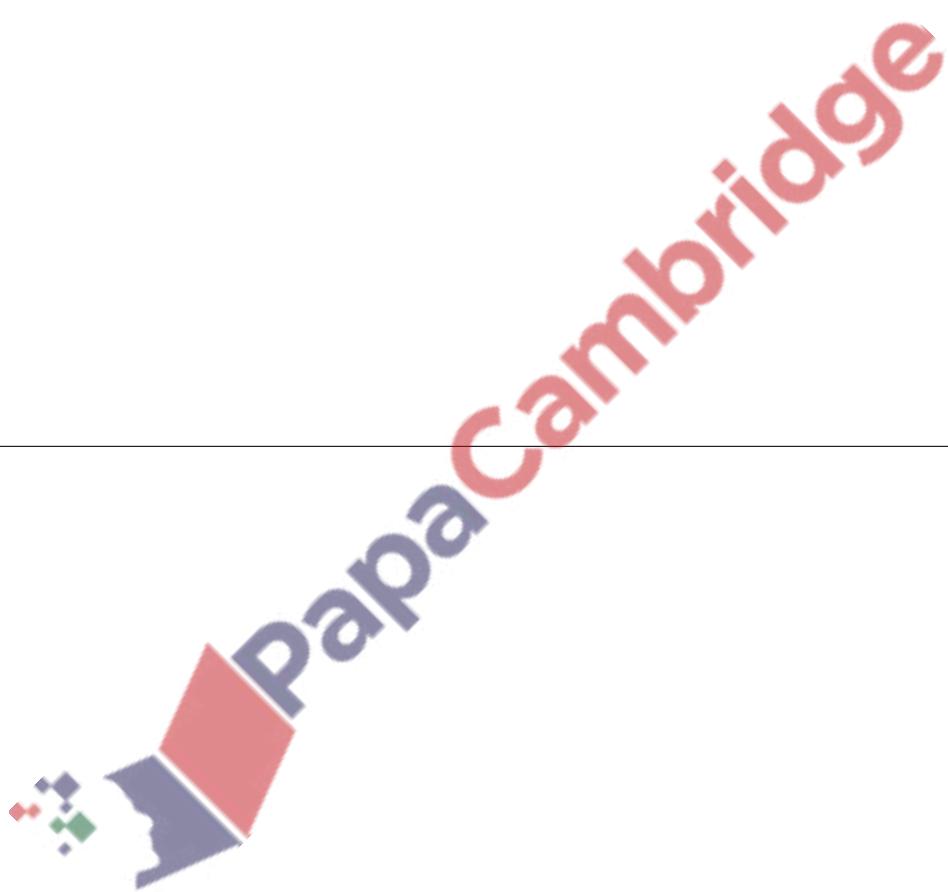
for $0^\circ < x < 360^\circ$.

[3]



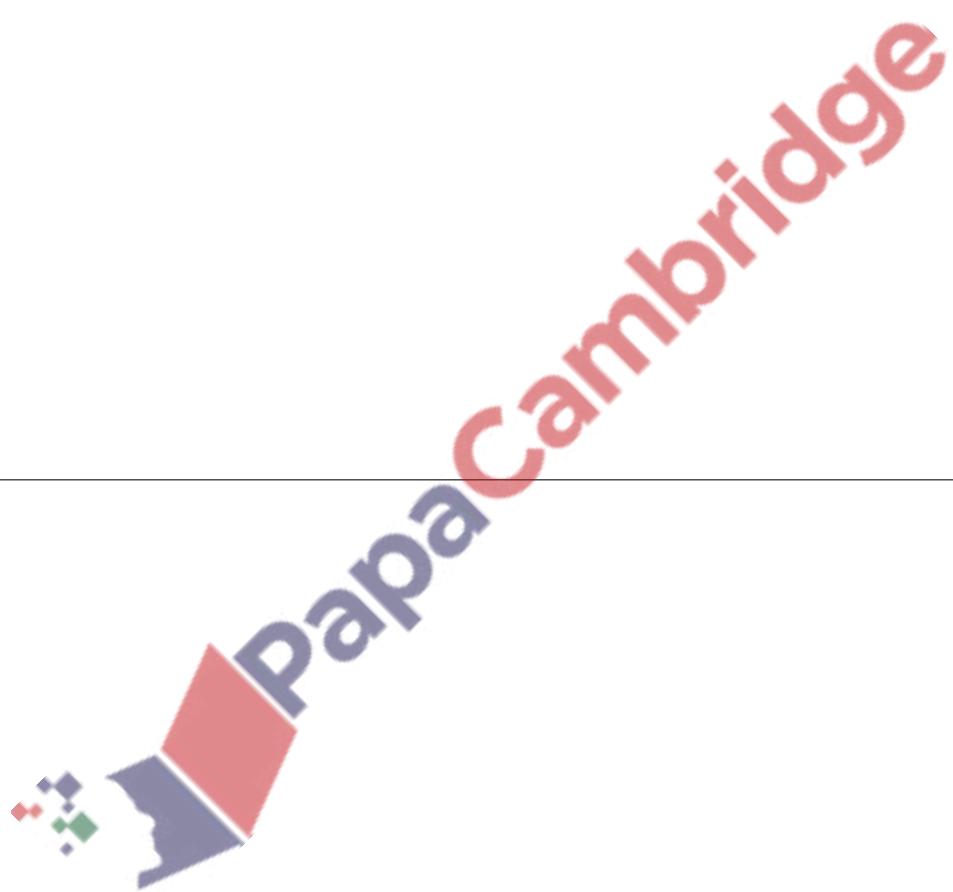
106. 9709_w16_qp_31 Q: 3

Express the equation $\sec \theta = 3 \cos \theta + \tan \theta$ as a quadratic equation in $\sin \theta$. Hence solve this equation for $-90^\circ < \theta < 90^\circ$. [5]



107. 9709_w16_qp_33 Q: 3

Express the equation $\cot 2\theta = 1 + \tan \theta$ as a quadratic equation in $\tan \theta$. Hence solve this equation for $0^\circ < \theta < 180^\circ$. [6]



108. 9709_s15_qp_32 Q: 4

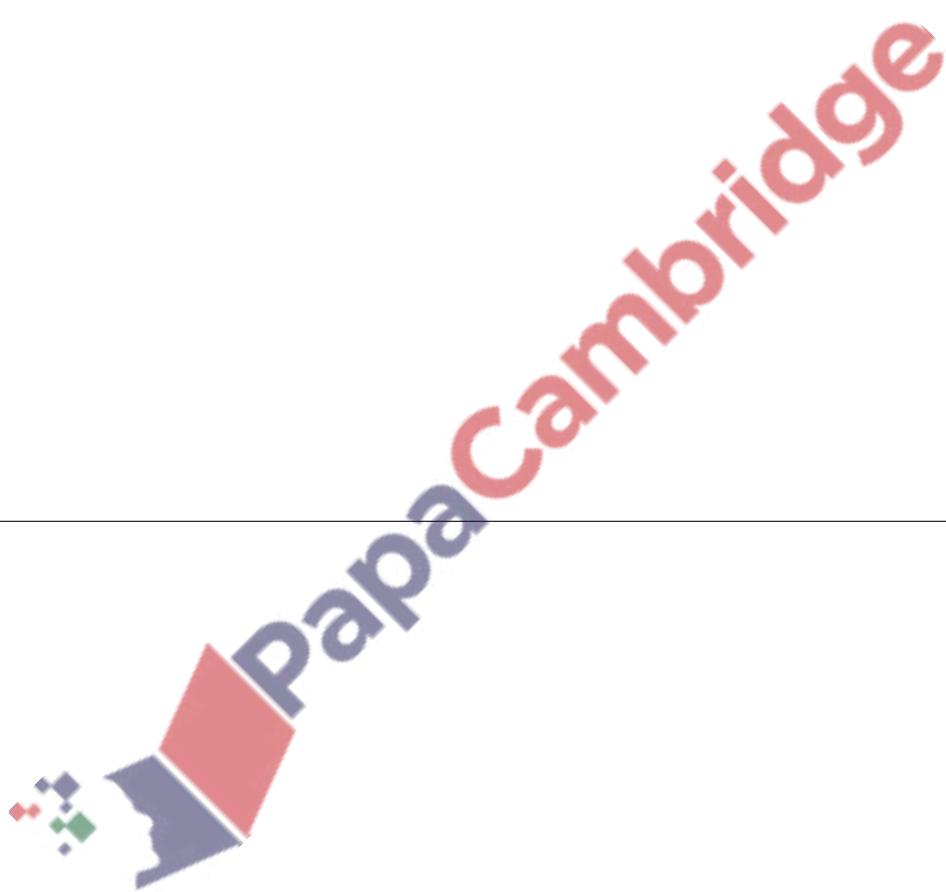
- (i) Express $3 \sin \theta + 2 \cos \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

- (ii) Hence solve the equation

$$3 \sin \theta + 2 \cos \theta = 1,$$

for $0^\circ < \theta < 180^\circ$.

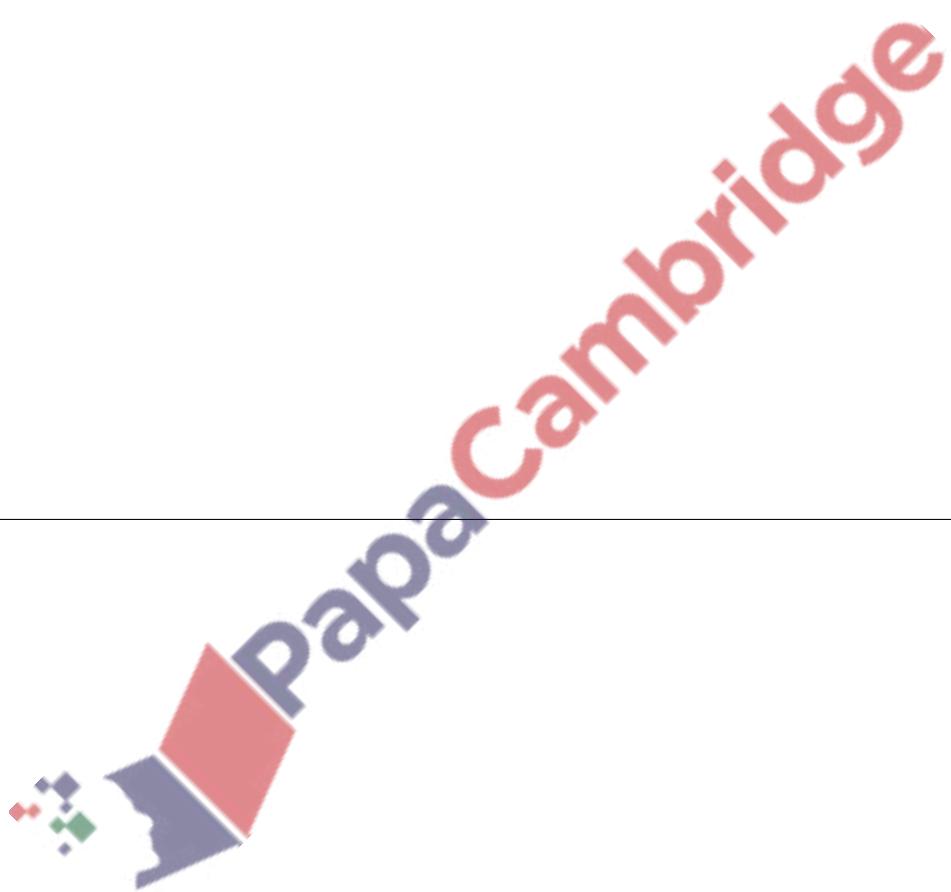
[3]



109. 9709_s15_qp_33 Q: 3

Solve the equation $\cot 2x + \cot x = 3$ for $0^\circ < x < 180^\circ$.

[6]



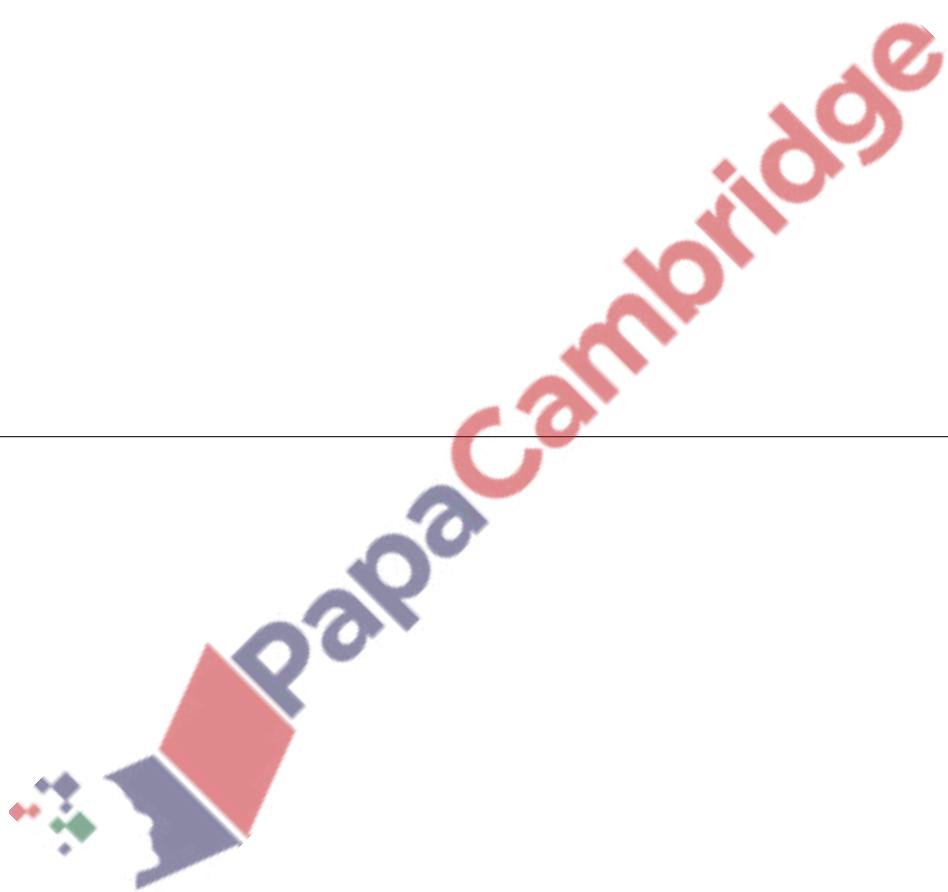
110. 9709_w15_qp_31 Q: 3

The angles θ and ϕ lie between 0° and 180° , and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan \theta + \tan \phi = 1.$$

Find the possible values of θ and ϕ .

[6]



111. 9709_w15_qp_33 Q: 6

The angles A and B are such that

$$\sin(A + 45^\circ) = (2\sqrt{2}) \cos A \quad \text{and} \quad 4 \sec^2 B + 5 = 12 \tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$.

[8]

